Chiral cavity quantum electrodynamics with coupled nanophotonic structures

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(Received 29 October 2018; published 19 November 2019)

Up to now it remains challenging to couple photons from a circularly polarized emitter into a photonic structure to simultaneously realize strong photon-emitter interaction and unidirectional propagation locked by local helicity of the optical mode at the nanoscale. In this paper we propose a unique approach that combines a photonic crystal and metallic nanoparticle structure to create nanocavities with both strong local-field intensity and high helicity. In this system the rate of circularly polarized photons emitting into the photonic crystal waveguide reaches 148γ0, which is one order of magnitude larger than that without the nanoparticle, and in the ultranarrow band-edge mode the linewidth of Rabi splitting spectra is about one-tenth of that with the nanoparticle only, both with ≈95% of photons propagating unidirectionally along the nanoscale waveguide. We suggest that our paper establishes a nanophotonic interface of chiral quantum electrodynamics for on-chip nonreciprocal quantum light sources, quantum circuits, and scalable quantum networks.

DOI: 10.1103/PhysRevA.100.053841

I. INTRODUCTION

Cavity quantum electrodynamics (CQED) studies light-matter interaction at a single quantum level [1,2]. With confined electromagnetic fields, optical mode volume and mode density determine the behavior of photon-emitter coupling. By compressing the optical mode into the region of several hundreds of micrometers, weak and strong couplings have been achieved in traditional CQED [3–5]. Recently, through reducing the optical mode volume into micro- and nanoscale, the CQED has succeeded in photonic crystals (PCs) [6,7], plasmonic nanocavities [8–11], whispering guided resonators [12], and various hybrid photonic systems [13–16], which paves the way to nanolasers, on-chip quantum devices, and scalable quantum networks [17]. Besides the ultrasmall mode volume, transversely confined light in photonic structures induces the local helicity of light, the handedness of which has a one-to-one relation to the propagation direction enforced by time-reversal symmetry [18–21]. If one puts a circularly polarized emitter into these structures, the propagation direction of emitted photons will be locked by the handedness of local spin, the so-called chiral photon-emitter coupling [22,23]. The propagation isolation of photons is able to avoid the signal disturbance and improve transmission efficiency, and hence can be utilized in various nonreciprocal quantum information components, e.g., chiral entanglement [24,25], quantum gates [26,27], switchings [28], isolators [29], and circulators [30].

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waveguide and can be directly used in on-chip nonreciprocal quantum light sources and quantum circuits.

The mechanism of chiral photon-emitter coupling in nano-CQED is described as follows. Different from previous chiral coupling at other scales \([12,30,35]\), it is difficult to use an individual nanostructure to realize both strong photon-emitter interaction and high directionality at the nanoscale. The realization of this dual function must rely on optical mode coupling between nanostructures. In traditional mode coupling, researchers generally focus on amplitude superposition after combination of different optical modes, while in order to achieve the above dual goals one must consider a real vector superposition, i.e., the superposition of both the amplitude and the direction between two vectors. Only if both strong local field and high helicity are simultaneously obtained, the above dual function can be realized at the nanoscale.

II. OPTICAL MODES OF COUPLED W1 PC AND AgNP STRUCTURE

Based on the above idea, we choose the coupled PC-AgNP structures to realize both strong photon-emitter interaction and high directionality at the nanoscale. First consider an individual AgNP. If the left-polarized incident light from the \( -z \) direction is used to excite a localized surface plasmon of a single AgNP, there is a strong local-field enhancement with nonzero helicity. Typically, the helicity of the \( z \) direction is defined as \( \frac{2 \text{Im}[E_x E_y^*]}{|E_x|^2 + |E_y|^2} \) \([36]\). As a result, when a \( \sigma_- \) emitter is located nearby the AgNP, there will be a large Purcell enhancement \([8–10]\). Secondly, if only a W1 PC is considered, the electric field is mainly located in the waveguide with antisymmetric local helicity distribution [Fig. 1(b)] when light from the \( x \) direction is incident on the W1 PC. If now putting a \( \sigma_- \) emitter into the area with red helical curves (such as at the point A or B in Fig. 5 in Appendix B), emitted photons will propagate along the left direction due to the spin-locked direction \([19,37–40]\), i.e., the propagation direction in all areas with red helical curves is connected [Fig. 1(d)]. More details are shown in Appendices A–C.

Then, through the vector coupling between the modes of the PC and AgNP, the typical optical mode possesses both large field enhancement and high helicity around the AgNP, and simultaneous high helicity inside the waveguide channel of the PC [Figs. 1(c) and 1(e)]. Different from only the PC structure, opposite helicity with a blue helical curve appears around the resonant AgNP, as shown in Fig. 1(c). If now one puts a \( \sigma_- \) emitter into the near-field region of the AgNP, both large chiral photon-emitter interaction and unidirectional propagation can be obtained. Specifically, the propagation direction is locked by the sign of local helicity where the emitter is, i.e., the emitted photons will propagate along the right direction [Fig. 1(e)]. Helicity details of the electric field are shown in Fig. 8 in Appendix D. Therefore, with coupled AgNP and PC structure, the chiral interaction is determined by local field while the propagation direction is locked by local helicity.
After the mode hybridization of the PC and AgNP, three optical modes arise; i.e., modes 1 and 3 are dipolar and quadrupolar modes of the AgNP, and mode 2 is the band-edge mode [Figs. 2(a) and 2(b)]. Here the W1 PC is chosen with the thickness of \( h = 0.84a \), hole radius of \( r = 0.29a \), and refractive index of \( n = 3.45 \), where \( a \) is the lattice constant, and computations are performed by the commercial COMSOL and finite-difference time-domain (FDTD) softwares. The process of model coupling is shown in Appendix D. When the radius \( r_m \) of the AgNP is 20 nm and \( a = 190 \) nm, there is a widening of the band gap of the PC due to strong coupling between two structures [Fig. 2(a)]. In contrast, as \( r_m = 7 \) nm and \( a = 170 \) nm, AgNP size is too small to affect the band of the PC [Fig. 2(b)]. After the parameter optimization, mode 3 in Fig. 2(a) situated at the guided band acts as a nanocavity to provide large Purcell enhancement and effective guidance of photons, while mode 2 in Fig. 2(b) is suitable to realize strong coupling due to its extremely narrow linewidth. In principle, by adjusting geometric and material parameters, the energy bands of the PC as well as the resonance of the plasmon nanoparticle can be modulated to arbitrary electromagnetic frequency.

III. NANOSCALE PURCELL ENHANCEMENT AND UNIDIRECTIONAL PROPAGATION WITH CHIRAL COUPLING

Then, Purcell enhancement of chiral coupling at the nanoscale is demonstrated. After the optimization of position and polarization, we put a \( \sigma_- \) emitter into the near-field region of the resonant AgNP [mode 3 in Fig. 2(a)] with \( \lambda = 658 \) nm and the distance of \( L = 2 \) nm. When it circles around the AgNP in the xy plane, i.e., \( \theta \) is changed from \( 0^\circ \) to \( 360^\circ \), the Purcell factor (here normalized total decay rate \( \gamma_{tot}/\gamma_0 \)) remains at very high values of 4500–4800 [Fig. 3(a)], among which the guided part \( \gamma_{WG}/\gamma_0 \) is 95–264 with extreme values corresponding to electric-field maxima of the AgNP. It is noted that the guided part \( \gamma_{WG}/\gamma_0 \) is one order larger than \( \gamma_{tot}/\gamma_0 \) (\( \approx 10 \)) of only the W1 PC even at the band-edge region [19,37]. There is also a correspondence between the maximum of directionality \( D_R \) and the minimum of decay rates, where \( D_{R \rightarrow L} = \frac{\gamma_{WG}/\gamma_0}{\gamma_{WG}/\gamma_0} \) and \( \gamma_{WG}/\gamma_0 \) is the energy power from the emitter into the right or left end of the channel. One can see the computation in Appendix E.

This correspondence can be explained as follows. If the \( \sigma_- \) emitter is placed at the positions with higher helicity, more emitted photons couple to the eigenmode with the same spin, leading to the maxima in the \( D_R \) curve. In contrast, if the emitter is located at the lower helicity region, such as for \( \theta = 45^\circ \), the directionality \( D_R \) is small, but two channels (to left and right directions) can transmit more photons, so the maxima of \( \gamma_{WG}/\gamma_0 \) appear. By comparing these curves with Fig. 8 in Appendix E, we prove that a strong local field leads to large Purcell enhancement and high local helicity results in good directionality. Moreover, the metallic loss of the AgNP only influences the Purcell enhancement obviously but not the local helicity, especially within the near-field region of the AgNP (see Appendices E and F). Also, dimer plasmon structure lacks a good directionality due to low helicity originating from the symmetry (see Appendix K).

To balance the inconsistence between the decay rate and directionality, in Fig. 3(b), the parameter of \( \theta = 180^\circ \) is chosen. At \( \lambda = 658 \) nm, both guided Purcell enhancement \( \gamma_{WG}/\gamma_0 = 148 \) and the directionality reach very high values. Especially, in the spectral range of 655–661 nm, \( \gamma_{tot}/\gamma_0 \) and \( \gamma_{WG}/\gamma_0 \) can reach 4200 and 110 and 98% of the guided photons propagate unidirectionally in Appendices G and H. Figures 3(c) and 3(d) depict electric-field and local helicity distributions when the electromagnetic mode is excited by the \( \sigma_- \) emitter with \( \lambda = 658 \) nm and \( \theta = 180^\circ \). As mentioned above, the existence of the AgNP changes the local helicity where the emitter is, thus the propagation direction is opposite to that without the AgNP.

Furthermore, by optical mode design, the coupled structure is capable of separating different wavelength photons into opposite directions. For example, in the above coupled structure, if the AgNP is substituted by a silver nanoblock, two optical modes will appear at different wavelengths. By optimizing the emitter’s position near the coupled nanostructure, one can obtain a strong localized field around the corner of the nanoblock but with an opposite sign of field helicity for two modes. If now putting different wavelength \( \sigma_- \) emitters into
the optimal position, besides a large Purcell enhancement of emitted photons, the guided parts $\gamma_{\text{WG}}/\gamma_0$ of Purcell factors are steered to the left or right direction separately with high directionality because of the opposite sign of local helicity at these two wavelengths. Therefore, the photons with the same circular polarization will propagate to different directions, which can be utilized to the on-chip routing single-photon source. For more details see Appendix I.

IV. RABI SPLITTING OF FLUORESCENCE SPECTRA AND UNIDIRECTIONAL PROPAGATION WITH CHIRAL COUPLING

Next, the vacuum Rabi splitting of energy levels in chiral coupling is demonstrated. We put the AgNP near the waveguide of the PC [inset of Fig. 4(a)] and demonstrate that the appearance of the band-edge mode is independent of the position of the AgNP in Appendix D. This mode essentially originates from the embedded AgNP that causes the guided mode of the PC to slightly shift below the cutoff region. The AgNP here has two roles, which not only works as a point mode of the PC to slightly shift below the cutoff region. The origin of the band-edge mode is independent of the symmetry of fluorescence spectra is broken. The energy exchange between the emitter and cavity photons is mediated by the coupling coefficients $g$ between the emitter and cavity modes. For both cases, $g$ decreases exponentially as an increment of $L$. For example, when $L = 2 \text{ nm}$ and $\theta = 257^\circ$, $g = 3.14 \text{ meV}$, $\kappa = 2.9 \text{ meV}$, and the decay rate $\gamma = 0.59 \text{ meV}$. In this case, the strong-coupling condition $g > (\kappa, \gamma)$ is satisfied [41].

By using the PYTHON toolbox (see Appendix J), fluorescence spectra of the $\sigma_+$ emitter coupled with the band-edge mode are obtained. Considering that the directionality $D_\theta$ at $\theta = 257^\circ$ is $94.7\%$ [Fig. 4(d)] and the point $M$ with the frequency detuning $\Delta = \kappa/2$ (between the emitter and cavity mode) is within the guided band of the PC [Fig. 4(a)], we choose the parameters of $\theta = 257^\circ$, $L = 2 \text{ nm}$, and $\Delta = \kappa/2$. It is found that the Rabi splitting in fluorescence spectra starts to appear at $\mu = 0.5 \text{ e nm}$ and becomes larger with its increment, while for $\mu = 1.0 \text{ e nm}$ there is an apparent energy exchange between the emitter and cavity photons [Fig. 4(e)]. Also, owing to the existence of detuning $\Delta$, the symmetry of fluorescence spectra is broken. The energy splitting and spectral linewidth are roughly coincident with $\sqrt{4\kappa^2 + \Delta^2} - \frac{\kappa - \gamma}{2}$ predicted by the dressed state theory [41,42]. The linewidth of fluorescence spectra is about one-tenth of that if only the AgNP. Hence, superior to the individual AgNP, the narrower linewidth of the band-edge mode leads to the earlier appearance of Rabi splitting with the same $\mu$. 

FIG. 3. Nanoscale Purcell enhancement and unidirectional propagation with chiral coupling. Normalized decay rates $\gamma_{\text{tot}}/\gamma_0$ and $\gamma_{\text{WG}}/\gamma_0$ and the directionality $D_\theta$ as a function of (a) $\theta$ and (b) $\lambda$. Here we use the quadrupolar mode of the AgNP [mode 3 in Fig. 2(a)] at $\lambda = 658 \text{ nm}$ to realize the chiral coupling and $L = 2 \text{ nm}$. (c), (d) The distributions of (c) electric field and (d) helicity for $\theta = 180^\circ$ when a $\sigma_+$ emitter excites the mode of the AgNP.
FIG. 4. Rabi splitting of fluorescence spectra and unidirectional propagation with chiral coupling. (a) Transmittance spectra of the PC and absorption spectra of the AgNP [mode 2 in Fig. 2(b)]. (b) The electric-field distributions of the band-edge mode with the insets of the region of $30 \times 30$ nm$^2$. (c) Coupling coefficient $g$ with varying distance $L$ for $\theta = 40^\circ$ and $257^\circ$ with $\mu = 1.0$ e nm. (d) Directionality $D_L$ of guided photons with varying $\theta$. (e) Fluorescence spectra of the quantum emitter as a function of $\omega - \omega_c$ with varying $\mu$. The inset shows $g, \gamma, \kappa$ dependence on $\mu$. (f) Helicity distribution corresponding to (b). (g) Electric-field distribution for $\theta = 257^\circ$ when the $\sigma_-$ emitter excites the band-edge mode. In (e) and (g), the detuning $\Delta = \kappa/2$.

In this case, the sign of electric field helicity around the AgNP is opposite to that of only the PC. So, if now putting the $\sigma_-$ emitter into its near-field region, the photons will be locked to the left direction [Fig. 4(g)]. Compared with the case of $\Delta = 0$, more photons can be transmitted because at the point $M$ the frequency of emitted photons lies in the guided band of the PC. For $L = 2$ nm and $\Delta = \kappa/2$, if letting the emitter walk a circle around the AgNP, the directionality $D_L$ reaches its maximum at $\theta = 40^\circ$ and $257^\circ$ [Fig. 4(d)]. Corresponding to these maxima, $\approx 95\%$ of guided photons propagate into one direction along the waveguide of the PC, which can be used in nonreciprocal quantum nanophotonic devices [23].

Finally, we address the fabrication possibility of our scheme. Nowadays, a single AgNP [8] and PC [40] can be fabricated by state-of-the-art nanotechnology. If the AgNP is embedded inside the PC, its oxidation or corrosion should be effectively prevented. Also, single emitters embedded in a PC waveguide have been realized through scanning tunneling microscopy [37]. The main challenge to realize efficient chiral coupling is how to precisely control relative positions between the emitter and AgNP, which may be solved by using an atomic force microscopy tip to move the nanoparticle after emitter position is fixed [43]. By applying a strong magnetic field and selecting laser frequency, we can generate two circularly polarized states $|+\rangle$ and $|\text{--}\rangle$ of the atoms [44] or quantum dots [26] to emit only $\sigma_+$ or $\sigma_-$ photons. Thus, it is possible to achieve our scheme experimentally in the near future.

V. SUMMARY

In summary, we have established a nanophotonic interface of chiral CQED by proposing the coupled photonic crystal and plasmon nanoparticle structure. The key element of chiral CQED is the joint action of nanoscale strong local field and high local helicity, which provides strengthened light-emitter coupling with good directionality of emitted photons. Using the basic idea presented here, other kinds of combined nanophotonic structures could be designed, such as a coupled nanowire and nanoparticle system. The study of chiral CQED greatly enriches the understanding of light-emitter
interaction at the nanoscale and provides a possible platform for on-chip quantum light sources, quantum circuits, and scalable quantum networks with nonreciprocal nature.

ACKNOWLEDGMENTS

We thank X. Hu and Y. Ao for helpful discussions. This work is supported by the National Key Research and Development Program of China under Grant No. 2018YFB1107200; by the National Natural Science Foundation of China under Grants No. 11525414, No. 11974032, and No. 11734001; and by the Key Research and Development Program of Guangdong Province under Grant No. 2018B030329001.

APPENDIX A: COMPUTATION MODULE

We use the commercial COMSOL multiphysics software to perform the simulations. The TE-like PC, containing $11 \times 12$ unit cells shown in Fig. 5(a), is placed in the middle of a three-dimensional module with the height of $h + 1 \mu m$ [Fig. 5(b)]. To minimize boundary reflections and form an infinite space, the scattering boundary condition is used to surround the module. An AgNP, the permittivity of which is taken from the experimental data [45], is embedded in the middle plane of the waveguide of the PC. Adding an incident plane wave propagating along the $x$ axis, or putting an oscillating point dipole inside the PC, modes of the coupled PC and AgNP structure can be excited.

The circular polarization of the light is represented by helicity, which is defined as [36]

$$C = \frac{|E_{LCP}|^2 - |E_{RCP}|^2}{|E_{LCP}|^2 + |E_{RCP}|^2},$$

(A1)

where $E_{LCP}$ ($E_{RCP}$) is the left (right) component of the electric field related to the basis. If we choose the basis of $\hat{x} + i\hat{y} \sqrt{2}$, $\hat{x} - i\hat{y} \sqrt{2}$, and $\hat{z}$, the helicity in the $z$ direction can be written as $C(z) = \frac{2 \text{Im}(E_x E_y^*)}{|E_x|^2 + |E_y|^2}$. In the same way, we can derive $C(x) = \frac{2 \text{Im}(E_y E_z^*)}{|E_y|^2 + |E_z|^2}$.

FIG. 5. (a) Schematic diagram of the W1 PC and (b) computation module of COMSOL software. One unit cell of the PC is marked within the white lines in (a). (c) Photonic band diagram for the period of $a = 190$ nm, the hole radius of $r = 0.29a$, the thickness of $d = 0.84a$, and the refractive index of $n = 3.45$. (d) The electric-field components $|E_x|$ and $|E_y|$ and (e) helicity distributions of the $z$ direction when $a/\lambda = 0.26$. (f) The electric-field distribution in the waveguide of the W1 PC. The electric fields are normalized by the maximum of $|E_y|$.
FIG. 6. Absorption spectra of the AgNP in a homogeneous medium with refractive index of \( n = 3.45 \), for the radius of (a) \( r_m = 7 \) nm, (b) \( r_m = 14 \) nm, and (c) \( r_m = 20 \) nm. The wavelength is normalized by \( a = 170 \) nm to compare with the case of the AgNP embedded in the PC waveguide. For the AgNP with different size, redshift of resonance wavelength occurs as \( r_m \) enlarges. (d) Purcell enhancement of the \( \sigma^- \) emitter near the AgNP embedded in a homogeneous medium. Here \( r_m = 20 \) nm, \( n = 3.45 \), and the distance between them is 2 nm. (e), (f) The electric-field distributions and (g), (h) helicity distributions of the dipole mode of a AgNP with \( r_m = 7 \) nm in the \( xy \) and \( yz \) planes. A left-handed polarized plane wave is used to excite the AgNP, which propagates along the \(-z\) direction.

and \( C(y) = \frac{2\text{Im}[E,E^*_y]}{|E_x|^2+|E_y|^2} \). From Eq. (A1), \( C = 1 \) represents the left-polarized light and \( C = -1 \) is the right-polarized light. In the COMSOL module, the left- and right-handed polarized plane waves along the \( z \) axis are given as \( \vec{E}_\pm = E_x \hat{x} + E_y \hat{y} = E_0 e^{\pm i \pi/2} \sqrt{2} \), where \( E_0 \) is the amplitude of the electric field and \( E_y \) is \( \pm \pi/2 \) out of phase with \( E_x \). Similarly, the left- and right-handed polarized emitters are set as oscillating point dipoles for \( \sigma_\pm = \frac{\mu \pm \mu e^{i \phi}}{\sqrt{2}} \), where \( \mu \) is the magnitude of the dipole moment. The transmittance spectrum of the PC is defined as \( I_t/I_0 \), where \( I_t \) and \( I_0 \) are the light intensity in the incident and exit surfaces of the PC. In this paper, the light is incident on the left end of the PC and the right end is used as the receiving surface to collect photons.

APPENDIX B: PHOTONIC BAND DIAGRAM OF THE PHOTONIC CRYSTAL

We compute the photonic band diagram of the W1 PC by a FDTD method with commercial software (Lumerical) [Fig. 5(c)]. To reduce the computation process, a two-dimensional module, where the thickness of the PC is represented by a modified refractive index \( n_{eff} \), is performed. Through boundary mode analysis in the COMSOL module, we obtain \( n_{eff} = 2.95 \). Compared with the transmittance spectra in Figs. 2(a) and 2(b), the band gap in Fig. 5(c) is coincident with that in the transmittance spectra obtained by COMSOL software. Typical distributions of the electric field and its helicity inside the waveguide of the PC are shown, respectively, in Figs. 5(d) and in Figs. 5(e) and 5(f).
FIG. 7. The transmittance spectra of the PC and the normalized absorption spectra of the AgNP with the radius of (a)–(c) $r_m = 7$ nm, (d)–(f) $r_m = 14$ nm, and (g)–(i) $r_m = 20$ nm. The parameters for $a = 170$–190 nm are computed to change optical modes hybridized by the PC and AgNP. Figures 2(a) and 2(b) in the main text correspond to (i) and (a), respectively, and the band gap of a PC without the AgNP is shown as the gray region. Mode 1 and mode 3 are the dipole and quadrupole modes of the AgNP and mode 2 is the band-edge mode corresponding to the K point in the band diagram of Fig. 5(c). By increasing the radius of the AgNP, the band-edge mode always exists and its gap width enlarges. Specifically, when $a = 180$ nm and $r_m = 7$ nm, mode 1 lies at the band edge of the PC so that modes 1 and 2 are almost overlapped.

APPENDIX C: THE ELECTRIC FIELD AND ITS HELICITY FOR THE MODES OF THE AgNP

In the following, we analyze the electric field and its helicity for the mode of the AgNP in a homogeneous medium with the refractive index of $n = 3.45$. The absorption spectra of the AgNP for different radii $r_m$ are shown in Figs. 6(a)–6(c). As $r_m$ enlarges, redshift in the resonance wavelength occurs. When $r_m = 20$ nm, $n = 3.45$, and the distance between the emitter and AgNP is 2 nm, Purcell enhancement of the $\sigma_-$ emitter near the AgNP embedded in the homogeneous medium is displayed in Fig. 6(d). Figures 6(e)–6(h) depict the electric-field and helicity distributions of the AgNP in the $xy$ and $yz$ planes, respectively. A left-handed plane wave of $\lambda = 625$ nm polarized in the $xy$ plane is used to excite the dipole mode of the AgNP with the radius of 7 nm. It is seen that the electric field in the $yz$ plane is similar to that of a linearly polarized dipole in the $y$ direction, while in the $xy$ plane the absolute value of the electric field is homogeneous around the AgNP. Note, the helicity around the AgNP in the $xy$ plane has an opposite sign against that of the excited light.

This phenomenon can be explained as follows. The equivalent dipole moment of the AgNP excited by a plane wave with $\vec{E} = [1, i, 0]E_0e^{-i(k_0z-\omega t)}$ is given as

$$\vec{p}_{\text{MNP}} = 4\pi \epsilon_0 \epsilon_b r_m^3 \Gamma (E_0 \hat{x} + iE_0 \hat{y}),$$

where $\Gamma = (\epsilon_m - \epsilon_b)/(\epsilon_m + \frac{n+1}{n-2} \epsilon_b)$ with the permittivity $\epsilon_m$ of the AgNP and $\epsilon_b$ of the host medium [46]. Take the dipole mode as an example, namely, $n' = 1, \Gamma = (\epsilon_m - \epsilon_b)/(\epsilon_m + 2\epsilon_b)$, and the resonant condition is $\epsilon_m = -2\epsilon_b$. Under the resonant condition, $[p_x, p_y] \propto [-i, 1]$, which has the same form as that of a $\sigma_-$ emitter the near field of which has positive helicity in the $yz$ plane. Similarly, the local polarization of the higher-order modes of the AgNP has the same sign as that of the dipole mode.

APPENDIX D: OPTICAL MODE COUPLING IN THE COUPLED PC AND AgNP STRUCTURE

The transmittance spectra of the PC and the absorption spectra of the AgNP with various radii $r_m$ and lattice constants $a$ are shown in Fig. 7. The gray region is the band gap of the
PC without the existence of the AgNP. We can see that the band gap is broadened as the radius $r_m$ of the AgNP enlarges. As in Figs. 2(a) and 2(b), mode 1 and mode 3 correspond to the dipole and quadrupole modes of the AgNP and mode 2 is a band-edge mode [43,47]. Here, lattice constant $a$ is changed to make the mode of the AgNP situate at different frequency regions of the PC, i.e., guided and band-gap regions. Take $r_m = 7$ nm, for example. As $a$ increases, the dipole mode of the AgNP moves from the band gap to the guided band of the PC. Comparing Figs. 7(a)–7(c), it is seen that narrow and sharp band-edge modes appear when the dipole mode of the AgNP locates at the band gap of the PC [Fig. 7(c)]. Redshift in the absorption spectra is also clearly shown as an increment of $r_m = 20$ nm.

For a single AgNP, a single PC, and the coupled AgNP and PC structure, the helicity distributions in the $xy$ plane are depicted in Figs. 6(f), 5(e), and 8(b), respectively. The signs of helicity between the excited AgNP and the external field are opposite [Fig. 6(f)]. If the AgNP is embedded in the PC, the hybrid mode has opposite spin against that without the AgNP [Figs. 8(b) and 5(e)]. The electric-field and helicity distributions of the hybrid mode within an area of $80 \times 80$ nm$^2$ are shown in Fig. 8. As seen in Fig. 8, the merit of the hybrid mode is illustrated by not only electric-field enhancement, but also the control and adjustment of the spin, which can be used to modulate both spontaneous emission of the emitter and the propagation direction of photons.

Finally, by adjusting geometric and material parameters, the energy bands of the PC as well as the resonance of the plasmon nanoparticle can be modulated to arbitrary electromagnetic frequency. For example, by reducing $a$ with fixed ratios $h = 0.84a$ and $r = 0.29a$, the transmittance spectrum can be moved to the UV region, and simultaneously the UV resonance of the Al nanoparticle [48] can be designed within the guided band or band edge of the PC. If now putting the Al nanoparticle into the waveguide of the PC and optimizing the parameters of the coupled system, the high directionality as well as large Purcell enhancement in the UV region should also be achieved.

APPENDIX E: COMPUTATION OF VARIOUS COEFFICIENTS

Three physical processes are included in the CQED systems: the coupling between the cavity photons and emitter, the decay of the emitter, and the cavity loss, the coefficients of which are labeled as $g$, $\gamma$, and $\kappa$, respectively. According to their relations, two typical regimes exist, i.e., weak coupling for $g \ll \gamma$, $\kappa$ and strong coupling for $g \gg \gamma$, $\kappa$ [3].

1. Weak coupling

In the weak-coupling regime, the emitter decays through three channels: guided along the waveguide $\gamma_{WG}$, radiating into the far field $\gamma_{free}$, and nonradiative loss $\gamma_{NR}$. Thus the total decay rate $\gamma_{tot}$ is equal to the sum of these decay rates, i.e., $\gamma_{tot} = \gamma_{WG} + \gamma_{free} + \gamma_{NR}$. The total normalized decay rate can be obtained from $\gamma_{tot}/\gamma_0 = W_{tot}/W_0$ [49], where $W_{tot}$ and $W_0$ are the total emitted energy power of an emitter in the coupled system and in vacuum, respectively. The energy power of the emitter in the COMSOL module is given by the surface integration of a nanosphere containing the emitter over the energy flows, which can be expressed by $W_{tot} = \oint_{\Sigma} \vec{S} \cdot d\Sigma$, and $\vec{S}$ is the Poynting vector on the nanosphere [15]. Similarly, the guided part along the waveguide is calculated by $\gamma_{WG}/\gamma_0 = W_{WG}/W_0$, where $W_{WG}$ is the energy power of summing over receiving surfaces, i.e., right and left surfaces of the module.

2. Strong coupling

The emitter-nanocavity coupled system in the strong-coupling regime can be represented by the Jaynes-Cummings model, where the Hamiltonian and dynamical equation are

$$H = \omega_0 \sigma_+ \sigma + \omega_a a^\dagger a + g (\sigma_+ a + a^\dagger \sigma)$$  \hspace{1cm} (E1)

and

$$\dot{\rho} = i[\rho, H]/\hbar + \frac{\kappa}{2} (2\sigma \rho a^\dagger - \sigma^\dagger a \rho) + \frac{g^2}{2} (2a \sigma a^\dagger - a^\dagger a \sigma),$$  \hspace{1cm} (E2)
where \( \omega_0 \) and \( \omega_e \) are the frequencies of the emitter and the nanocavity, respectively; \( \sigma (\sigma^+) \) is the lowering (raising) operator of the emitter; and \( a (a^+) \) is the bosonic annihilation (creation) operator of the nanocavity. The physical process in this system is that the emitter and nanocavity exchange energy with coupling coefficient \( g \), and, simultaneously, photons also decay because of the atomic decay and Ohmic loss of the cavity, with coefficients labeled as \( \gamma \) and \( \kappa \). In the following, we will show how to compute these coefficients by means of COMSOL software.

### a. Coupling coefficient \( g \)

Following the equations and method in Refs. [16,50],

\[
\hbar R = \mu E_a
\]

where \( E_a \) is the electric field of the nanocavity corresponding to a single excitation and can be written as

\[
E_a = \frac{\bar{E}}{\sqrt{W_{\text{loc}}}}.
\]

where \( \bar{E} \) is the excited electric field of the nanocavity, \( \sqrt{W_{\text{loc}}} \) denotes the number of photons with energy \( \hbar \omega_e \), and \( W \) is the total energy of the cavity mode which is calculated by energy density integration in the whole space [16]:

\[
W = \frac{1}{2} \int \frac{\partial}{\partial \omega} [\omega \text{Re}(\varepsilon(\omega))]_{\omega=\omega_0} [\bar{E}]^2 dV + \frac{1}{2} \int \mu_0 |\vec{H}|^2 dV.
\]

(E4)

In the following, we will verify the correctness of the coupling coefficient calculation. As mentioned above, \( W \) is the total energy of the cavity mode which is calculated by energy density integration in the whole space. But in fact, when the integration range is large enough, the energy obtained is close to the total energy \( W \) and the resulting coupling coefficient approaches the true value. Here, the considered nanocavity is a hybrid cavity formed by the interaction of the AgNP and the PC [corresponding to mode 2 in the main text of Fig. 2(b)]. Electric-field distribution of this nanocavity is shown in Fig. 9(a). In addition to strong local fields near the nanoparticles, there is also a large field distribution near the PC waveguide. Three different extracted energy regions are marked with white lines in Fig. 9(a). Their sizes are 6a × 4.33a × (h + 100 nm), 8a × 4.33a × (h + 100 nm), and 10a × 5.20a × (h + 100 nm) for block1, block2, and block3, respectively. It is worth mentioning that energy is not only localized in the PC layer, but also a small amount of energy exists in the air layer above and below the PC layer. Thus based on those three regions, add 50-nm-thick air layers above and below to form three new regions. Their sizes are 6a × 4.33a × (h + 100 nm), 8a × 4.33a × (h + 100 nm), and 10a × 5.20a × (h + 100 nm) for block1, block2, and block3, respectively. In particular, the regions of block2 and block3 are shown in Figs. 9(c) and 9(d). Finally, comparison of coupling coefficients obtained by various ranges of extracted energy is shown in Fig. 9(b). These coupling coefficients are very close, which means that these ranges are large enough that coupling coefficients are very close to true value. In the main text, the coupling coefficients are obtained by energy integration in the range of block2.
The cavity loss $\kappa$ is the full width at half maximum (FWHM) of its extinction spectra, including the scattering and absorption of the cavity. Because the FWHMs of scattering and absorption spectra are almost the same [51], $\kappa$ is derived from the absorption spectrum here. In the COMSOL module, the mode of the AgNP is excited by a nearby oscillating point dipole. By volume integrating the AgNP over the power density, the resistive loss $W_{\text{NR}}$ is obtained. $\kappa$ of the AgNP in the homogeneous environment with refractive index $n = 3.45$ can be obtained in Figs. 6(a)–6(c). Using the same procedure, $\kappa$ of the AgNP in the PC can be obtained, too.

c. Decay rate $\gamma$

Decay rate $\gamma$ here indicates the atomic decay rate to the modes other than the cavity mode, i.e., except for the nonradiative decay caused by the loss of the AgNP. Therefore, $\gamma$ is equal to the total decay rate minus the nonradiative part, i.e., $\gamma = \gamma_{\text{tot}} - \gamma_{\text{NR}}$, which can be obtained by $\gamma = \frac{W_{\text{nr}} - W_{\text{rad}}}{W_0} \gamma_0$ and $\gamma_0 = \frac{\omega_0^2 \mu^2}{3 \pi \epsilon_0 hc^3}$ [52].

APPENDIX F: COMPUTATION OF DIRECTIONALITY FOR PHOTON PROPAGATION

We compare the directionality with different areas of the receiving surface of the module. Figure 10(a) depicts the directionality $D_R$ with the size of the receiving surface of $3.5\sqrt{3}a \times (h + 1 \mu m)$, $1.5\sqrt{3}a \times (h + 1 \mu m)$, and $6.5\sqrt{3}a \times h$, respectively. Because the photons lose the lock of propagation direction in the free space, directionality reduces when the surrounding environment is involved. However, the differences in these three cases are small and the wavelengths of the maxima in these three cases are not changed. So, in the main text, the area of $3.5\sqrt{3}a \times (h + 1 \mu m)$ is used. In this case, more than 90% of the radiative photons are coupled to the waveguide of the PC [Fig. 10(b)].

To investigate the effect of metallic loss to the Purcell enhancement and directionality of photon propagation, we give the absorption spectra, directionality $D_R$, and normalized decay rate $\gamma_{\text{CW}}/\gamma_0$ for different loss rates [Fig. 11(a)]. By keeping the real part $\text{Re}[\epsilon_m]$ of the dielectric constant as the original value for each wavelength, we artificially increase the imaginary part $\text{Im}[\epsilon_m]$ by a factor of 1, 2, and 3. It is found that increasing the loss of metal material does not obviously affect the peak position and directionality, but does affect the Purcell enhancement. The reason is that the local electric field determines the Purcell enhancement, which is greatly influenced by the metallic loss, while the directionality of photon propagation comes from the local helicity, which is not greatly affected by the metal loss, especially within the near-field region of the AgNP [Figs. 11(c)–11(e)]. Other parameters in Fig. 11 are the same as those in Fig. 3.

APPENDIX G: PURCELL FACTOR AND PROPAGATION DIRECTIONALITY WHEN AN EMITTER COUPLES TO ONLY THE PC

First, we discuss the Purcell enhancement of a circularly polarized emitter coupled to a PC. Some experimental and theoretical works show that the Purcell factor in the slow-light region of the PC can reach 10 as well as the directionality over 90% [19]. Here, we put a right-handed polarized emitter $\sigma_-$ in the PC with $y = 50\text{ nm}$ [Fig. 12(a)] and $y = 0$ [Fig. 12(b)], then obtain the decay rate of the emitter and directionality of the emitted photons. As shown in Fig. 12, the total decay rate in both cases has an increment at the band edge of the PC, namely, in the slow-light region [Fig. 5(c)]. In this region, the gradient of the energy band is very small, so the group velocity of the photons is also slow [19,40]. That is why the emitter has large Purcell enhancement at the
band edge of the PC. In our system, the maximum $\gamma_{WG}/\gamma_0$ is less than 15 with the directionality of 92% in the case of $y = 50$ nm. While the helicity is $\approx 0$ when $y = 0$, the numbers of photons propagating to right and left directions are almost the same, i.e., losing the direction lock of photon propagation.

**APPENDIX H: POSITION OPTIMIZATION OF THE Emitter AND AgNP IN THE COUPLED PC AND AgNP STRUCTURE**

To obtain high directionality of photon propagation, in the main paper, we optimize the parameters of the system by...
FIG. 13. (a) Electric field and (b) its helicity of the $\sigma^-$ emitter in the coupled PC and AgNP structure, for $r_m = 20\text{ nm}$, $y = 50\text{ nm}$, and $\theta = 45^\circ$. The photons propagate to two sides along the waveguide of the PC, leading to small directionality $D_{\lambda}$. The symmetry of helicity in both propagating channels is not broken compared with that with $\theta = 180^\circ$ in the main text. Other parameters are the same as those in Fig. 3.

selecting the positions of the emitter around the AgNP. The helicity where the emitter is placed should be large enough to ensure a unidirectional photon propagation. For Fig. 3, the case that $\theta = 180^\circ$ is chosen, the directionality is 98%. In contrast, when $\theta = 45^\circ$, the directionality is only 66% due to the small helicity, the electric-field and helicity distributions of which are depicted in Fig. 13.

Moreover, the case that the emitter is placed at the planes of $z \neq 0$ is discussed. For example, the normalized decay rate and directionality of the right-handed polarized emitter $\sigma^-$ at the top of the AgNP with a distance of 2 nm are shown in Fig. 14. The total decay rate and its guide part are increased less than those in the plane of $z = 0$ and the directionality is only 63.8% at $\lambda = 658$ nm due to the low helicity of 0.6. So the results in Fig. 3 are optimized by the position of the emitter.

As we did for Fig. 4, to find the narrowest linewidth of the band-edge mode [mode 2 in Fig. 2(b)], we change the position of the AgNP from point 1 to point 7 in Fig. 15(a). After comparing the full widths at half maximum of all hybrid modes, we finally place the AgNP at point 6 with the linewidth of $\kappa = 2.9$ meV [Figs. 15(b)–15(h)].

For a $\sigma^+$ emitter, to verify the same Purcell enhancement and an opposite photon propagation as those of the $\sigma^-$ emitter, we compute the normalized decay rates $\gamma_{\text{tot}}/\gamma_0$ and $\gamma_{\text{WG}}/\gamma_0$ and the directionality $D_{\lambda}$ of the emitted photons as a function of $\theta$ and $\lambda$ [Fig. 16]. Other parameters are the same as those in Fig. 2. As expected, the directionality and decay rates have the same maximal values due to the symmetry. From the helicity distribution, we can see that the symmetry of the propagation to the right direction is broken, which is opposite to that of the $\sigma^-$ emitter in Fig. 3(d).

### APPENDIX I: PARAMETER OPTIMIZATION IN THE COUPLED PC AND Ag NANOBLOCK STRUCTURE

A coupled PC and silver nanoblock structure is capable of separating the photons with different wavelengths into opposite propagation directions. As shown in Fig. 17(a), if the AgNP is substituted by a silver nanoblock with the length of 25 nm and height of 10 nm, its optical modes appear at the wavelength of 705 and 639 nm, respectively. Because the helicity has opposite signs around the nanoblock for the two modes [Figs. 17(b) and 17(c)], according to the principle...
FIG. 15. (a) Schematic diagram of the AgNP with $r_m = 7$ nm located in different positions of the PC with $a = 170$ nm. Supposing the coordinate of point 1 is $(0, 0)$, the coordinates of points 2–7 are $(-0.5a, 0)$, $(-0.5a, 50)$ nm, $(0, 50)$ nm, $(0, 0.5\sqrt{3}a)$, $(0, 0.75\sqrt{3}a)$, and $(-50, 0.75\sqrt{3}a)$. (b)–(h) The absorption spectra of the band-edge mode for various AgNP positions marked in (a). The linewidth $\kappa$ of the AgNP is optimized by selecting its position. In the main text, point 6 with $\kappa = 2.9$ meV is chosen.

FIG. 16. Normalized decay rates $\gamma_{\text{tot}}/\gamma_0$ and $\gamma_{\text{WG}}/\gamma_0$ of a $\sigma_+$ emitter and the directionality $D_L$ of the emitted photons as a function of (a) $\theta$ and (b) $\lambda$ for $a = 190$ nm, $r_m = 20$ nm, and $L = 2$ nm. In (a), the maxima of $\gamma_{\text{WG}}/\gamma_0$ generally corresponds to minima of $D_L$ and vice versa, while $\gamma_{\text{tot}}/\gamma_0$ keeps in a high range of 4600–4800. $\theta = 0^\circ$ is chosen in (b). When $\lambda = 658$ nm, Purcell enhancement reaches $\gamma_{\text{tot}}/\gamma_0 = 4700$ and $\gamma_{\text{WG}}/\gamma_0 = 148$, where 98.4% of emitted photons propagate to the left direction. (c, d) The distributions of (c) the electric field and (d) its helicity when a $\sigma_+$ emitter excites the quadrupolar mode of the AgNP. From the helicity distribution, the symmetry of the propagation is broken, which is opposite to that of the $\sigma_-$ emitter in Fig. 3(d).
FIG. 17. (a) Absorption spectra of the silver nanoblock embedded in the waveguide of the PC with $a = 180$ nm. Here, the nanoblock with length of 25 nm and height of 10 nm is placed at $y = 50$ nm. (b), (c) The electric-field and helicity distributions for the nanoblock at (b) $\lambda = 639$ nm and (c) $\lambda = 705$ nm.

FIG. 18. (a) Schematic diagram of the coupled PC and Ag nanoblock structure. Normalized decay rate $\gamma_{WG}/\gamma_0$ and directionality as a function of the emitter’s position, for (b) $\lambda = 639$ nm and (c) $\lambda = 705$ nm. The maximal values of the directionality are 85 and 99% corresponding to $\lambda = 639$ and 705 nm, respectively. Here, the nanoblock is 25 nm in length and 10 nm in height, $a = 178$ nm, $h = 0.84a$, and $r = 0.29a$. 

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FIG. 19. (a) Normalized decay rates $\gamma_{\text{tot}}/\gamma_0$ and $\gamma_{\text{WG}}/\gamma_0$ of a $\sigma_-$ emitter and (b) directionality as a function of lattice constant $a$ for $\lambda = 639$ and 705 nm. When $a = 178$ nm, $\gamma_{\text{tot}}/\gamma_0$, $\gamma_{\text{WG}}/\gamma_0$, and directionality are 8079, 307, and 99% for $\lambda = 639$ nm, respectively, and $\gamma_{\text{tot}}/\gamma_0 = 13\,461$, $\gamma_{\text{WG}}/\gamma_0 = 69$, and $D_L = 85\%$ for $\lambda = 705$ nm. Here, for both wavelengths, the emitter is placed at the position with the highest directionality shown in Fig. 18.

FIG. 20. (a) Normalized decay rates and (b) directionality $D_{\text{R}}$ or $D_L$ as a function of $\lambda$ for two emitters. The emitter is placed in the position with the maximal helicity for each wavelength. (c), (d) The electric field in the waveguide at (c) $\lambda = 639$ nm and (d) $\lambda = 705$ nm. Other parameters are the same as those in Fig. 14.

FIG. 21. (a) Directionality $D_L$ of emitted photons dependent on $\theta$. The AgNP with $r_m = 7$ nm is placed at point 6 of Fig. 15. Directionality $D_L$ reaches the maximum when $\theta = 12^\circ$ and 190$^\circ$ with the value of 89.6%. (b) Fluorescence spectra of the quantum emitter as a function of $\omega - \omega_c$ with varying dipole moment $\mu$. The parameters $g, \gamma, \kappa$ in dependence on $\mu$ are shown in (c). Here, $L = 2$ nm and the detuning $\Delta$ between the nanocavity and emitter is zero.
of chiral coupling, the photons with these two wavelengths emitted from the emitter with the same polarization should be guided to opposite directions. To prove this point, we first optimize the parameters of the PC. The basic requirement here is to transmit photons with the wavelength of 639 and 705 nm simultaneously, so the choice of lattice constant $a$ should make sure that the wavelengths of both photons are within the guided band of the PC, namely, $a/\lambda$ should be within the range of 0.25–0.3. Thus, after the computation, $a$ corresponds to a range of 176–191 nm.

Figure 18 depicts normalized guided decay rate and directionality of a right-handed polarized emitter $\sigma_-$ coupled to an Ag nanoblock with the length of 25 nm and the height of 10 nm. The lattice constant $a$ is 178 nm. The distance between the emitter and the surface of the nanoblock is fixed at 2 nm. And the position of the nanoblock is (0, 50 nm, 0) (supposing the origin of the coordinate is at the center of the PC). After moving the emitter from region A, via region B, to region C, it is seen that the directionality $D_L$ can easily reach 90% at region B for $\lambda = 639$ nm. Especially, the maximal $D_L$ is 99% with $\gamma_{WG}/\gamma_0 = 68$, while for $\lambda = 705$ nm the photons are guided to the right direction and the maximal directionality $D_R = 85\%$ with $\gamma_{WG}/\gamma_0 = 346$. Comparing Fig. 18(b) with Fig. 18(c), we find the maximal helicity appears at the corner of the nanoblock at $\lambda = 639$ nm, but deviates from the corner at $\lambda = 705$ nm, which are coincident with the positions with maximal directionality. Also, the localized electric field at $\lambda = 705$ nm is much stronger than that at the other wavelengths, so a larger decay rate is obtained.

Furthermore, to find out the influence of lattice constant $a$ of the PC, we investigate $\gamma_{WG}/\gamma_0$ of the emitter and directionality of emitted photons at $\lambda = 639$ and 705 nm (Fig. 19).
FIG. 23. (a) Normalized absorption spectra of an Ag dimer excited by x and y polarized plane waves. The resonant wavelengths are 652 and 648 nm, respectively. Electric-field and helicity distributions in the cases of (b) y polarized plane wave and (d) x polarized plane wave. The radii of the AgNPs are 20 nm with a gap distance of 4 nm.

According to the results in Figs. 18(b) and 18(c), the emitter should be placed at the positions where the directionality reach their maxima for two wavelengths. When $\lambda = 178$ nm, the directionality $D_R = 99\%$ for $\lambda = 639$ nm and $D_E = 85\%$ for $\lambda = 705$ nm, which are both maximal values in their own cases [electric-field distributions are shown in Figs. 20(c) and 20(d)]. Correspondingly, at $\lambda = 639$ nm, the normalized decay rate $\gamma_{WG}/\gamma_0 = 307$, while, at $\lambda = 705$ nm, $\gamma_{WG}/\gamma_0 = 69$ [Figs. 20(a) and 20(b)].

**APPENDIX J: FLUORESCENCE SPECTRA OF THE QUANTUM EMITTER**

The PYTHON toolbox is performed to derive the resonance fluorescence spectrum of the CQED system by Fourier transformation electric intensity $(E^{-}_{R}(t), E^{+}_{R}(t))$, with the expression of $S(\vec{r}, \omega_0) = \frac{1}{2} \text{Re} \int d\tau (E^{-}_{R}(\vec{r}, \tau) E^{+}_{R}(\vec{r}, \tau)) e^{i\omega_0\tau}$ [52,53]. By increasing the dipole moment of the emitter, Rabi splitting appears in the fluorescence spectra [Figs. 4(e) and 21]. When the transition frequency of the emitter is resonant with the frequency of the nanocavity, the peaks of Rabi splitting are symmetric (Fig. 21). By contrast, asymmetry of the spectrum occurs as the emitter frequency deviates from that of the nanocavity mode [Fig. 4(e)]. In the following, we present the code and show how to obtain the resonance fluorescence spectrum by means of the PYTHON toolbox.

By importing the QUTIP package [53], some quantum-mechanics operations, such as commutation and solving equation of motion, can be easily done. Note, the operators and Hamiltonian should be expressed by the matrix with the dimension of 2. As shown in Fig. 22, “omega_c” and “omega_a” are the frequencies of the nanocavity and the emitter, with detuning of $\chi/2$; “kappa,” “gamma,” and “g” are the cavity loss, the decay rate of the emitter, and the coupling coefficient between the nanocavity and the emitter. We let the thermal environment be characterized by an average particle expectation value of $\langle n \rangle = n_{th}$. The dynamics of the system can be written as $\dot{\rho} = i[H, \rho] / \hbar - \Gamma(\rho)$. Here the Hamiltonian and the system decay $\Gamma$ are represented by “H” and “c_ops.” Then the spectra are calculated by the correlation function using the mesolve solver in the QUTIP package, and then obtained by Fourier transform. Finally, we give an example of the PYTHON program for the resonance fluorescence spectrum of $\mu = 1.0 \, \text{e nm}$ in Fig. 4(e) (see the screen shot in Fig. 22).

**APPENDIX K: DIMER PLASMON NANOSTRUCTURE**

Dimer plasmon nanostructures are widely used to enhance the Purcell factor because of the ultralarge electric-field enhancement within the nanoscale gap [54,55]. To investigate the directionality of photons in coupled PC and AgNP dimer structure, we first explore the optical modes of only the dimer structure with the radius of 20 nm and the gap distance of 4 nm. The x- and y-polarized plane waves are used to excite the AgNP dimer [inset of Fig. 23(a)], where x-polarized light polarizes parallel to the axis of the dimer and y-polarized light is perpendicular to it. Corresponding to these two excitons, the resonant wavelengths of 652 and 648 nm are obtained (Fig. 23). Figures 23(b) and 23(c) show their electric-field and helicity distributions. It is seen that x-polarized light is able to excite the gap surface plasmon, while by using y-polarized light the electric field distributes around the dimer. Importantly, in both cases, the helicity within the gap is almost zero. If the incident light is changed to a circularly polarized wave, the obtained mode is the superposition of these modes, thus the helicity within the gap is still almost zero.

Then a coupled PC and AgNP dimer structure is considered, where a right-handed polarized emitter $\sigma_+$ is put at the center of the gap. Both the dimer and the emitter are embedded in the middle plane of the PC [Fig. 24(a)]. We obtain the decay rate $\gamma_{WG}/\gamma_0$ of the emitter and the directionality $D_R$ of photon propagation with varying the wavelength of the emitter [Fig. 24(c)]. It is seen that the resonant wavelength of the dimer is 652 nm, which means that the coupled mode is mainly excited by the electric-field component that is parallel...
FIG. 24. (a) Schematic diagram of the coupled PC and AgNP dimer structure. (b) Schematic diagram of the polarization ellipses of the electric fields around the AgNPs. (c) Normalized decay rate $\gamma_{WG}/\gamma_0$ of a $\sigma_-$ emitter and the directionality $D_L$ of the emitted photons as a function of $\lambda$. At the resonant wavelength of $\lambda = 652$ nm, $\gamma_{WG}/\gamma_0$ reaches maximum while $D_L = 49.5\%$. (d) Electric-field and (e) helicity distributions of the AgNP dimer. The parameters of the PC are the same as those in Fig. 5 and those of the AgNP dimer are the same as those in Fig. 23.

...to the axis of the dimer. At $\lambda = 652$ nm, the decay rate of the guided part $\gamma_{WG}/\gamma_0$ is 225 with $D_L = 49.5\%$. To explain the results, Figs. 24(d) and 24(e) depict the electric-field and helicity distributions of the dimer within an area of $120 \times 60$ nm$^2$, where the electric field in the gap is greatly enhanced while the helicity is almost near zero, leading to the large Purcell enhancement but a bidirectional photon propagation.

The reason for bidirectional propagation is described as follows. The symmetry structure generally destroys the helicity of the local field [Fig. 24(b)]. For the nanoparticle dimer, the electric field for each particle has the same handedness, so within the gap region the polarization ellipses of the electric fields counteract each other, leading to near-zero helicity. In general, this result is suitable for general dimer nanostructures, such as the bowtie structure and nanosphere dimer.


Movable high-Q nanoresonators realized by semiconductor nanowires on a Si PC platform, Nat. Mater. 13, 279 (2014).


